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## Encyclopedia of Mathematics and Its Applications

## STATEMENT

The ENCYCLOPEDIA OF MATHEMATICS was organized in 1974 by a group of mathematicians and scientists brought together by the resolution to make the resources of mathematics available, accessible, and understandable to the wide public of engineers and scientists.

Until now, the public has been satisfied to provide financial and social support to the mathematician, whose intellectual wanderings and occasional waywardness were forgiven by virtue of his contributions to the common good. It was once a widely-held opinion that in the mathematics of today one would find the basic concepts for the science of tomorrow. Hilbert space and spectral theory were the forerunner of quantum mechanics; tensor calculus was the handmaiden of general relativity. But nowadays, the results of mathematics often remain distant from their potential users, guarded with pride behind a barricade of notational complexity flying the banner of uncompromising rigor. The pressure of current events shows the danger of this position.

The existence of only five regular solids in space was a fact before it become a theorem. Other statements, which had had a difficult birth in the form of theorems, eventually came to be taken as facts. Standards of rigor, notation, methods of proof vary from one generation to the next; but the factual achievements of mathematics remain, ready to produce the spark of understanding, to be put to further application, to create the wonder of conceptual unity.

The need for gifted middlemen in mathematics is now more deeply felt than ever. The abundance of results, together with the elegance of today's mathematical presentation, have made the mathematician reluctant to regard his achievements with the otherwise motivated eyes of the technologist. But technologists, awed and made shy by this dazzling but distant firework display, are more unable than reluctant to benefit from the insights of mathematics. This concern, together with our conviction that all science sooner or later must prove itself by becoming mathematical, is the moving force behind this ENCYCLOPEDIA.

Specialists in all branches of mathematics will be asked to write thorough, clear presentations, complete and definitive insofar as possible, accessible to workers in fields as far removed from their own as the subject permits. Each such presentation is published as a volume loosely related to the others. Style of presentation, choice of contents, and interdependence of the subjects of various volumes are left to each author's decision; nor does the order of appearance of subjects follow a plan. Any such plan would inevitably be limited by the availability of an author.

The volumes of the ENCYCLOPEDIA published so far have met with a measure of success, and can be a guide for the ones to come. An encyclopedic presentation need not be a linear, axiomatic development. The singling out of the central results and, what is more, the explanation of those relevant examples which lead to familiarity and are thus indispensable to understanding should take precedence over the zeal for completeness. Less important facts and applications may be given in the form of exercises with references to the sources, or in notes and comments which relate, motivate, and help bring the subject to life. Each volume should also be considered as a textbook, suitable for self-study.

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The essentials of a theory are not always to be found in the statements of theorems. They may be found in arguments in the core of a proof, in algorithms, in examples, even in diagrams. We cannot dispense with rigor in a mathematical presentation; nevertheless, the kind of rigor demanded by some subjects may be at variance with the antiseptic style initiated by Euclid and carried to fruition by Landau and by Chevalley. A physicist's rigor differs from a topologist's, and a computer scientist may not be concerned with the fine points of algebra. A rigidly deductive exposition is more often dictated by the fear of error than by the wish for clarity.

In bygone days, a mathematician could pride himself on a self-contained presentation of his theory, shining by its depth, and surviving thanks to the approval of his peers. Nowadays, another kind of writing is also called for, not only because the readership of mathematics is shrinking but, more importantly, because of the changed understandings of the roles of mathematics and of mathematical reasoning. Mathematics, like poetry, music, philosophy, and most intellectual achievements, is unfortunately neither an economically self-sustaining enterprise, nor a self-sufficient creation. The illusion of self-sufficiency is the forerunner of the recurring threat of extinction.

Today, unlike yesterday, a writer on the theory of operators in Hilbert space who does not tell about quantum mechanics is not just neglecting to mention the most substantial application of his field: he is simply not telling the truth. A writer on group representations who does not go to the length of working out tables of characters and transforms — dull as they may be — is short-changing his readers. The reader expects courtesy and attention even in a printed text, and will no longer stand the agony of deciphering a mathematical text, pencil and paper at hand. He will lay down the book if the notation does not agree with common usage, even if the proposed alternative has been dictated by the most scrupulous criteria of logical consistency.

Clarity and accessibility, indispensable as they are in this enterprise, are however not all. Any encyclopedia is expected to be a repository of results of lasting — we are tempted to say permanent — value. The reader should turn to a volume not only as a rewarding experience; he should feel compelled to give the book a permanent place on his shelf as the definitive reference, frequently consulted and ever helpful.

The choice of subjects to be presented is restricted. Topics enjoying a vogue that may be ephemeral will have to prove their worth. On the other hand, forgotten theories that are raring to come back into the society of active mathematics may benefit from an exposition taking into account the experience of years of neglect. Subjects that straddle different fields are probably the timeliest, particularly if they hold promise for a breakthrough of mathematics into a newer science.

May this ENCYCLOPEDIA help realize, however modestly, the spirit of unity of science that was Leibniz's life dream. His vision has now reached at least the possibility of fulfillment.

The Editors

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# Encyclopedia of Mathematics and Its Applications

Each volume of the ENCYCLOPEDIA is a monograph on pure and applied mathematics. The presentation of the subject is left largely to the author's taste and preference, subject to the following broad guidelines:

INFORMATION FOR AUTHORS

- (1) The subject of each monograph should be of current interest.
- (2) The presentation should make the subject accessible to non-specialists; it should be written for easy consultation and understanding by scientists barely acquainted with the subject, though conversant with related kinds of mathematics.
- (3) The exposition, insofar as possible, should be of permanent value. The subject should be thoroughly surveyed, and a comprehensive guide to the literature should be given.
- (4) The notation chosen should follow established usage whenever possible.
- (5) Less important results may be summarized in the form of exercises at the end of each chapter, thereby making the monograph available as a textbook on the subject.
- (6) Each volume is usually preceded by one or more introductions written by the Section Editor or by other specialists in the field, outlining the present state of the subject, its applications, its prospects for the future, and its relations to neighboring branches of mathematics. Such introductions may run to greater lengths than customary.
- (7) Examples and applications should be unsparingly given, especially when they illuminate the theory. They should be chosen whenever possible from examples treated in the literature or relevant in applications.

The ENCYCLOPEDIA is subdivided into Sections, each of which is supervised by a member of the Editorial Board. Manuscripts may be recommended for publication by any member of the Editorial Board, or may be submitted directly to the Publisher. Each manuscript will be refereed by the Section Editor, by the Editor, and/or by the Publisher. On the basis of the volumes published so far, it may fairly be stated that most completed manuscripts are subject to revisions, which at times may be extensive and thorough. A manuscript is accepted for publication by the Section Editor and by the Editor, and a letter of acceptance is sent to the author by the Publisher on the recommendation of the Editor.

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